Mapping and Inference in Analogical Problem Solving – As Much as Needed or as Much as Possible?

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Abstract

There exist many computational models of analogy making, which are based on different assumptions about the cognitive processes which are responsible for successful transfer from a base to a target domain. In most models, mapping of the base structure to the target is seen as the core of analogy making. We believe that the way in which this mapping is realized is a crucial aspect by which the different models can be distinguished. More specific, we identified systematic (as much as possible) versus pragmatic (as much as needed) approaches to mapping as critical assumption by which the computational models can be discerned. In case of pragmatic mapping, only such aspects of the base domain are carried over to the target which address the current focus of interest. In contrast, in case of systematic mapping, the greatest relational structure which contains the aligned parts of base and target is carried over. In consequence, in systematic approaches, many aspects of the target problem are inferred at one time and available for future use, while in pragmatic approaches new questions concerning the target domain may require that mapping and inference must be performed again. We conducted an experiment in the domain of physics to investigate which type of mapping is realized by human problem solvers and got significant evidence for systematic mapping.

Keywords: Analogical problem solving; systematicity principle; analogical inference; computational models; problem solving.

Introduction

Most approaches to analogy agree that the discovery of a common structure between base and target – that is, mapping – is central to analogical processing (Gentner & Markman, 2006; Gentner, 1983; Holyoak & Thagard, 1995; Hofstadter, 2001). What parts and how much of the sub-structure of the base is mapped with the target determines success of analogical transfer (Novick & Holyoak, 1991; Schmid, Wirth, & Polkehn, 2003). Although mapping is realized in all computational models of analogy making, the way it is performed differs significantly. Some dimensions in which models can be compared with respect to the realization of the mapping process are: Restriction to first-order mappings or allowing for higher-order mappings, restriction to one-to-one mapping or allowing for non-isomorphical mappings, guiding mapping by structural constrains only or allowing for additional semantic and pragmatic constraints.

The structure mapping engine (SME, Falkenhainer, Forbus, & Gentner, 1989), one of the most influential models of analogical mapping an transfer, is based on first-order (objects from the base can be mapped to differently named objects in the target, identity mapping for function and relational symbols), one-to-one, structurally guided mapping. Other models deviate in one or more aspects from these assumptions. For example, LISA (Hummel & Holyoak, 1997) allows for mapping of different relations. Because mapping possibilities thereby become more complex, semantic and pragmatic constraints are used in addition to structural ones. The Analogy via Abstraction (AvA) model (Weller & Schmid, 2006; Schmid, 2005) allows for mapping of relations with different numbers of arguments, that is, it gets rid of the one-to-one assumption. Gentner and Markman (2006) present empirical evidence that people base their matches on conceptual relations rather than pure graph isomorphs, that is, their data support the first-order one-to-one assumption of SME.

In our opinion, the strongest assumption of SME is that mapping is based on structural constraints only, whereby structural mapping is guided by the systematicity principle. By this principle, an alignment of structural information that is associated by higher-order relations is preferred over the alignment of single, isolated structures. Due to the systematicity principle it is possible to transfer information from the base system without explicitly mapping them on corresponding structures in the target. It is only necessary that such information is connected to a higher-order relation which was aligned during the mapping process. In a problem-solving scenario the systematicity principle implies that, for a given problem statement, not only such parts of the base which are relevant to produce an answer are transferred to the target but that all inferences about the target domain are drawn which can be derived by systematic carry-over from the base domain. In the following, we will call this type of systematic mapping and inference the "as much as possible" principle.

While the AvA model differs from SME because it allows many-to-one and higher-oder mapping, it is also based on the "as much as possible" principle. In contrast to SME,

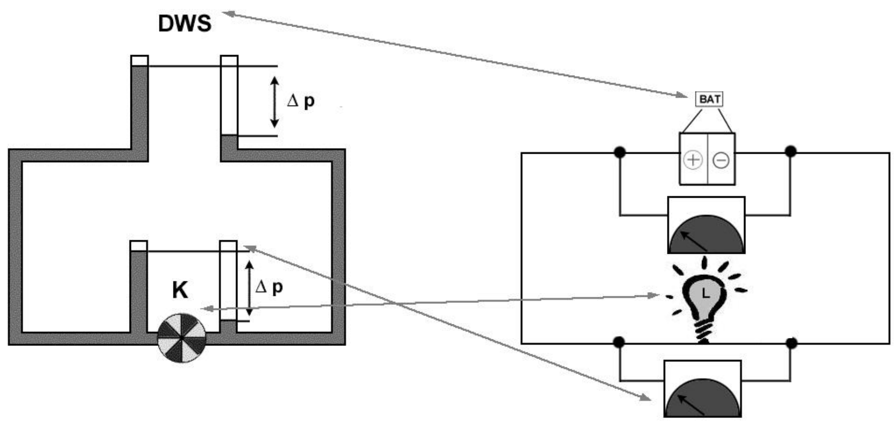


Figure 1: Circuit models of the electrical current analogy

the structural alignment is not realized as a simple structure mapping but as a structure mapping via the common abstract structure of base and target domain. Both AvA and SME in a first step identify the structural commonalities between base and target. In contrast to SME, AvA explicitly builds an abstract structure which subsumes both domains. This abstract structure governs the mapping of entities from base to target. Thereby, search for suitable mappings is omitted and entities are matched with respect to the roles they play in the common structure. In a problem-solving scenario modelled by AvA the results may be the same as in SME: One can use already familiar knowledge from the base to solve problems in the target. When solving a problem there is not only such knowledge transferred from the base which is necessary to solve the current problem but also information which is connected to the greatest common subgraph of already mapped structures.

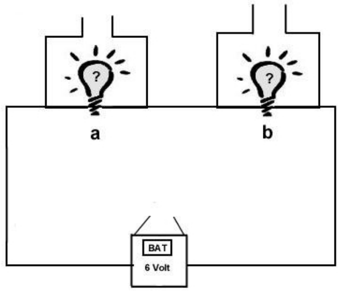
In contrast to SME and AvA, in most other models (e.g., Lisa, Hummel & Holyoak, 1997; AMBR, Kokinov & Petrov, 2001; Copycat, Hofstadter, 2001) mapping is realised by pragmatic rather than systematic methods, which we will call the "as much as needed" principle. That is, in a problem solving context, only such aspects of the base are transferred to the target which are necessary to produce a solution to the current problem statement. If new problem statements have to be addressed afterwards, it can be necessary to perform a new mapping and transfer to obtain a solution.

We conducted an experiment in which we directly confronted the "as much as possible" and the "as much as needed" assumption in the domain of physical problem solving. Physical problems are highly suitable because the underlying knowledge is highly structured and, at the same time, problems are sufficiently complex such that inferences cannot easily be drawn spontaneously without prior knowledge. Since the problem scenarios have to be constructed very carefully to provide for the possibility to transfer only as much as needed versus as much as possible knowledge from the base to the target, in the following section, we will describe the domain in some detail. Afterwards we will report our experiment and we will conclude with a final evaluation and further work to be done.

Analogical Problem Solving in the Domain of Physics

A well-known analogy in physics is the water flow to electrical current flow analogy (Gentner & Gentner, 1983). In figure 1 the simple circuit models together with the mapping of the basic components of both problems is shown. Both water circuit (base domain) and electrical circuit (target domain) consist of three main components with analogical functionalities. In the water circuit a double-water-column (DWS) causes water flow if there is a pressure difference (labelled Δp) between the two columns. The second component is a gyroscope (labelled K) which indicates the strength of water flow. The higher the flow rate of the water becomes, the faster the gyroscope revolves. The third component is needed to measure the pressure difference over the gyroscope. It is therefore called pressure-difference-analyzer. Like the double-water-column it is built up by two water columns around the gyroscope. In electric circuits the double-water-column is replaced by a simple battery which causes electricity flow because of an electric charge difference between its two poles. Such a charge difference is shown by the needle deflection of a voltage-analyzer. The common role of DWS and battery is to be the actuator in the circuit. In the electric flow domain, the gyroscope is replaced by a bulb which indicates the current strength. The brighter the bulb sparkles the higher the current strength is. Again, the voltage-analyzer is used to measure the fall of voltage over the bulb. The common role of gyroscope and bulb is to be the consumer in the circuit.

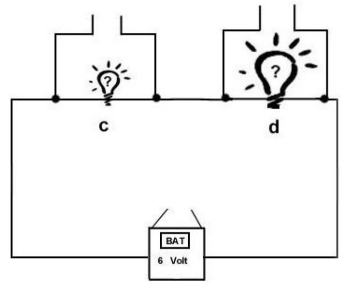
The main parameters of a (water or electrical) circuit are current strength I, voltage U (analog to pressure difference Δp) and resistance R (due to bulb or gyroscope). There is a multiplicative relation between these parameters which is given by Ohm's law: $U = R \times I$. This principle defines that voltage is directly proportional to resistance ($U \sim R$) and cur-



(Circuit 1)

The circuit is provided with a battery with a voltage of 6 Volt. Bulb *a* and *b* sparkle with the same intensity. What is the ration between the voltages that you can measure over bulb *a* and bulb *b*? (i) Voltage over bulb *a* is higher; (ii) Voltage over bulb *b* is higher; (iii) Both voltages are equal; (iv) I don't know.

Figure 2: Voltage problem, addressing U and I (type-1)



(Circuit 2)

In contrast to circuit 1 (fig. 2), bulb *a* has been replaced by a bulb *c* with less resistance and bulb *b* has been replaced by a bulb *d* with higher resistance. Current strength remains unmodified. What is the ratio between resistance in circuit 1 and circuit 2? (i) Resistance is smaller in circuit 2; (ii) Resistance is higher in circuit 2; (iii) Resistances in both circuits are equal; (iv) I don't know.

Figure 3: Resistance problem, addressing R and I (type-2)

rent strength ($U \sim I$), whereas current strength and resistance are indirect proportional ($I \sim \frac{1}{R}$). This law also holds in the water current domain. To keep things simple, in the following, we will always refer to the circuit parameters in the well-known terminology of electrical currents, even if we refer to the base domain.

If the (water/electrical) current circuit contains more than one actuator or more than one consumer one has to distinguish between serial and parallel connections. In our experiment, we only considered serial connections with one actuator and one to three consumers in row. If the circuit contains more than one consumer and consumers are serially connected, voltage and resistance are fragmented over every single consumer. Partial voltages or resistances then add up to the voltage or resistance of the system. Following the rules for serial circuits, the current strength I in a serial circuit is constant. Given either partial voltages V_i or partial resistances R_i, the other, unknown, parameter can be calculated using Ohm's law: $R = R_1 + \ldots + R_n$ can be derived from $U = U_1 + \ldots + U_n$ because Ohm's law indicates that voltage U and resistance R are proportional. One can measure the highest decrease of voltage over the highest resistance. Therefore, partial voltages U_i can be calculated by $U_i = R_i \times I$, if information about partial resistances R_i is provided. This works the same way for calculating R_i from U_i.

To contrast the "as much as needed" and the "as much as possible" assumption, it is necessary to provide a base domain knowledge structure consisting of a higher-order relation and two distinguishable first-order structures which are connected via the higher-oder relation. Ohm's law represents a higher-order relation connecting I, U, and R in a specific way. A first type of first-order relations consists of the adding up of partial voltages U_i over a series of serial connected consumers. A second type of first-order relations consists of the adding up of resistances R_i of series of serial connected consumers. The current strength I is a constant, equal at every possible point of measurement in the circuit. Figure 2 gives an example of a target problem addressing only the first type of relations. Figure 3 gives an example for a target problem addressing the second type of relations.

Experiment

In our experimental setting, subjects learn all concepts of serial rows for the water circuit domain. Afterwards, they have to solve problems in the electrical current domain, such as given in figures 2 and 3. The problems are designed in such a way that they allow for both the "as much as needed" and the "as much as possible" mapping and inference principle. Depending on which principle applies to analogical problem-solving, different results can be expected for transfer: If the "as much as needed" principle holds, only that information should be transferred to the target that is essential to solve the actual problem. Following that assumption, given a first problem concerning only type-1 (voltage) relations, only knowledge concerning voltage should be transferred to the target. That is, if subjects afterwards are confronted with a problem concerning type-2 (resistance) relations, mapping and inference must be performed again. The same should hold, if resistance problems are presented first and voltage problems afterwards.

In contrast, if the "as much as possible" assumption holds, when solving a problem concerning type-1 relations knowledge about both voltage (type-1) and resistance (type-2) should be transferred from the water circuit to the electrical circuit because both structures are connected via Ohm's law. In consequence, if the "as much as possible" assumption holds, it should suffice to solve a target problem of either type to transfer all principles contained in serial circuits in only one step.

Method

Materials The experiment was done on a computer, consisting of a tutorial for the base problem and a collection of voltage target problems as well as a collection of resistance

Table 1: Experimental design

	Experimental		Control	
	E_1	E_2	C_1	C_2
Tutorial type	Water Circuit		Electric Circuit	
Testing time 1	type-1	type-2	type-1	type-2
Testing time 2	type-2	type-1	type-2	type-1

target problems. In the tutorial, all concepts were described by spoken voice together with written information and animated graphical illustrations. All relevant facts were summarized in a table. Furthermore, subjects had to answer multiple choice questions about the base domain. If an answer was wrong, the correct answer was given together with an explanation. The tutorial finished when a subject answered all questions correctly. The target problems consisted of electrical circuit problems where answers were also given by multiple choice as shown in figures 2 and 3. Answers and answer times were written in a log-file. The complete material (in German) is given in Wiese (2007).

Design and Procedure The experimental design is summarized in table 1. Two experimental groups received a tutorial about the water circuit domain. Two control groups received a tutorial giving the same information in the electrical current domain. Afterwards, for all groups, problems of the electrical current domain were presented. One experimental group (E_1) and one control group (C_1) first worked on voltage problems (type-1) and afterwards on resistance problems (type-2). The other experimental group (E_2) and the other control group (C_2) worked first on resistance problems (type-2)and afterwards on voltage problems (type-1).

If the "as much as possible" hypothesis holds, solution times should decrease from the first to the second problem solving episode (testing time 1 to testing time 2) for the experimental groups (without an increase in erroneous answers). To take into account that voltage questions (type-1) and resistance questions (type-2) might take different time effort to answer, the time differences of interest are the ones between type-1 questions answered at testing time 1 (E_1) and type-1 questions answered at testing time 2 (E_2) and the differences between the type-2 questions at first (E_2) and the second (E_1) testing time.

The control groups were introduced because the experimental sessions needed about one hour of time and we had to take into account exhaustion and habituation effects. That is, we had to consider that either solution times increase from the first to the second problem solving episode due to the demanding nature of the problems, or that solution times decrease because subjects get used to the procedure of solving multiple choice problems. Therefore to support the "as much as possible" hypothesis, it is not sufficient to test a main effect for solution time from the first to the second problem solving episode. Instead, the interaction between tutorial-type (experimental vs control) and problem solving episode is tested.

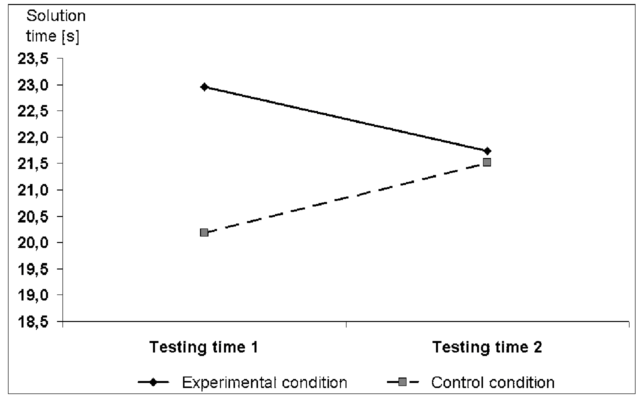


Figure 4: Interaction of testing time and tutorial-type wrt average solution times

Participants Seventy students of different social science programs at the University of Bamberg participated in the study. Seven subjects had to be excluded from analysis because they already knew the physical laws that had to be learned. The distribution of subjects over the four groups was: E_1 15 subjects (8 female, 7 male), E_2: 16 subjects (10 female, 6 male)), C_1: 16 subjects (12 female, 4 male), and C_2: 16 subjects (12 female, 4 male)). The average age was 23 (range 19 until 35). On average the last mark in physics was 3.0 (corresponding to a *C*). 87 percent of the subjects did not have contact with physics in the last five years.

Results and Discussion

To control that changes in solution times between episodes are not correlated with changes in error rates, differences in solution rates were tested. The average accuracy was about 85 percent. There were significant main effects of tutorial-type (experimental vs. control groups, $p = .026$) and problem-type ($p < .001$) with higher errors rates for the control conditions and for resistance (type-2) problems but no significant interactions. For analyses of solution times only such problems were included which subjects solved correctly. Tests were done with regression analysis based on the general linear model with dummy coding for subjects.

The main result is given in figure 4. The empirical data confirm the "as much as possible" hypothesis. The interaction between testing time and tutorial-type is significant ($p = .041$). Additionally, there are main effects for tutorial-type ($p = .019$) and problem-type (type-1 vs. type-2, $p < .001$). The main effect for testing time is not significant ($p = .929$). Additionally there are significant interactions between problem-type and tutorial-type ($p = .045$) and between testing time and problem-type ($p = .013$). The overall interaction between tutorial-type, testing time and problem-type is not significant ($p = .374$).

The main effect for tutorial type is due to the fact that subjects in the experimental groups need significantly more time to solve the problems at testing time 1 (see figure 4).

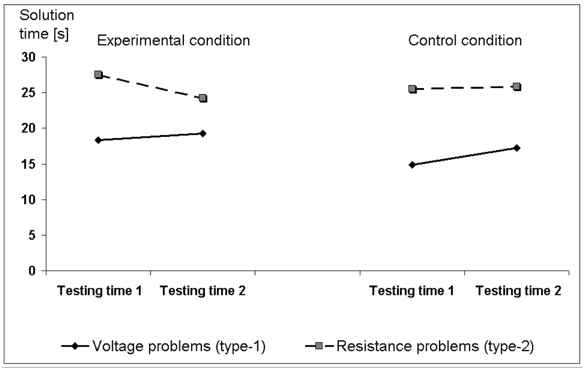


Figure 5: Average solution times for the electric circuit problems

This result supports our assumption that at testing time 1 subjects in the experimental groups needed to perform analogical mapping and transfer from the water circuit to the electrical circuit domain. In contrast, subjects from the control groups only needed to apply the concepts learned in the tutorial to the problems. At testing time 2, however, experimental and control groups need similar times for solving the problems. Again, this fact is supporting the "as much as possible" hypothesis: Subjects have inferred knowledge about all concepts learned in the water circuit domain to the electrical circuit domain during testing time 1 and therefore, at testing time 2 could exploit this knowledge without further need of mapping and transfer. The slight increase in solution time for the control groups can be explained by exhaustion effects – which are overridden by the inference advantage of the experimental groups.

A closer look at the results shows that the interaction effect of testing time and tutorial-type, i. e., the confirmation of the "as much as possible" hypothesis, is due to the resistance (type-2) problems only (see figure 5): While solution times for the experimental groups decrease significantly between the first an the second episode for resistance problems, this is not true for voltage problems. Although, voltage and resistance problems were designed as similar as possible with respect to complexity, subjects had more effort to solve the type-2 problems (concerning resistance). In average they needed 8.4 seconds more time to answer resistance questions than to answer voltage questions with a rate of 79 percent of correct answers for resistance questions and 94 percent for voltage questions. Therefore, we must assume that resistance problems are harder to solve than voltage problems.

On the other hand, subjects have a high profit of analogous transfer from the water circuit to electrical circuit domain by solving resistance (type-2) problems first and no profit when solving voltage (type-1) problems first. This could be due to the fact that measurement of resistance is realized very similar in the water circuit and the electrical circuit domain – in both cases it is shown as an obstruction of flow. Voltage, on the other hand, is shown by a pressure-difference analyzer in the water circuit and as voltage-analyzer in the electrical circuit. Therefore, analogical mapping might be more easy for type-2 (resistance) problems and only under that condition mapping and inference were fully realized by the subjects. However, since the threefold interaction of testing time, tutorial type and problem-type is not significant, there is no reason to assume that the processes of analogical mapping and transfer vary due to the problem-type. A more plausible explanation is that exhausting effects of different strength did obscure the transfer effect: Since resistance (type-2) problems had been of higher difficulty, it is possible that subjects who solved these problems at testing time 1 showed more exhaustion at testing time 2 (when solving the voltage problems) and therefore the transfer effect from resistance problems to voltage problems was shadowed by the exhaustion effect for this condition. To test this assumption, a new experiment with less demanding material should be performed.

Conclusion and Further Work

The results of our experiment support the assumption of "as much as possible" mapping and transfer which is inherent in the systematicity principle of SME and in our Analogy via Abstraction model. That is, it is plausible for computational models of analogy to greedily map as large a structure as possible from base to target and to transfer as much of knowledge from the base to the target as possible. More specifically, we could show that, if two disjunct relational structures (for pressure differences resp. voltage and for resistance) are connected via a higher-order relation (Ohm's law), analogical problem solving which addresses only problems of one relational structure triggers transfer of the complete structure. In consequence, in later problem solving episodes in the target domain, the relevant knowledge from the base is already available for the target and the new problems can be solved without performing a new mapping and transfer from base to target.

Due to the complexity of our physical problem domains the effect was somewhat obscured by exhaustion effects. Therefore, we plan to conduct a further experiment with the same design but with much more simple material. To prevent that problems in the simple domain can be solved in a trivial way, without analogical reasoning, we plan to construct artificial domains. Furthermore, it might be interesting to replicate the study with mathematical problems or programming problems. In addition, it might be interesting to present the problems to SME and some suitable alternative model to get a clearer understanding of how the "as much as possible" respectively "as much as needed" principles are realized in these systems.

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